Research Foundation & Evidence of Effectiveness for FASTT Math™

In the twenty-first century, solid math skills are a prerequisite for school achievement and success in the workplace. Although national math achievement has improved slightly, too many students still do not have basic math skills. According to the 2003 National Assessment of Educational Progress (NAEP) results, 23 percent of fourth graders and 32 percent of eighth graders performed at Below Basic levels in mathematics. Many of these struggling students lack fluency in basic math facts. With *FASTT Math*™, a new software program from Tom Snyder Productions, students gain math fact fluency in addition, subtraction, multiplication, and division. Developing fluent recall of the basic facts allows students to focus on more complex computations, problem solving, and higher-order math concepts.

FASTT Math uses research-validated methods to provide systematic instruction and continuous practice to help students automatically recall and understand math facts. The program uses adaptive technology to offer each student a customized learning experience based on their individual needs.

This paper introduces the research-based principles behind *FASTT Math* and explains the program's unique features that enable student success. Finally, the paper presents efficacy data that shows the dramatic improvement in math fact fluency for students who received instruction using the *FASTT Math* approach.

Research Foundation & Evidence of Effectiveness for FASTT Math™

Research Foundation & Evidence of Effectiveness for FASTT Math™

Table of Contents

PURPOSE & RATIONALE

PURPOSE

The goal of the *FASTT Math* program is to develop in students the ability to retrieve the answers to basic math facts from memory, both accurately and fluently. Fluently, here, means automatically and with understanding. (The *FASTT Math* program helps to identify and remediate those students who lack a conceptual foundation of the operations and math fact quantities.) Educators and cognitive scientists agree that the ability to recall basic math facts fluently is necessary for students to attain higher-order math skills. Through an adaptive program of systematic instruction and practice, *FASTT Math* helps students to abandon the use of inefficient strategies for determining the answers to basic facts, such as finger counting, and helps them develop the capacity to retrieve the basic facts from memory quickly and effortlessly.

RATIONALE

FASTT Math uses the FASTT system (*Fluency and Automaticity through Systematic Teaching with Technology*), based on nearly two decades of research on the development of mathematical fluency in math-delayed and non-math-delayed children. The rationale for this program is that all human beings have a limited information-processing capacity. That is, an individual simply cannot attend to too many things at once. Grover Whitehurst, the Director of the Institute for Educational Sciences (IES), noted this research during the launch of the federal Mathematics Summit in 2003:

> "Cognitive psychologists have discovered that humans have fixed limits
> on the attention and memory that can be used to solve problems. One
> way around these limits is to have certain components of a task become
> so routine and over-learned that they become automatic." (Whitehurst, 2003)

The implication for mathematics is that some of the sub-processes, particularly basic facts, need to be developed to the point that they are done automatically. If this fluent retrieval does not develop, then the development of higher-order mathematics skills—such as multiple-digit addition and subtraction, long division, and fractions—may be severely impaired (Resnick, 1983). Indeed, studies have found that lack of math fact retrieval can impede participation in math class discussions (Woodward & Baxter, 1997), successful mathematics problem solving (Pellegrino & Goldman, 1987), and even the development

of everyday life skills (Loveless, 2003). And rapid math fact retrieval has been shown to be a strong predictor of performance on mathematics achievement tests (Royer, Tronsky, Chan, Jackson, & Marchant, 1999).

Research by LaBerge and Samuels (1974), Lesgold (1983), and Torgesen (1984) support the notion that fluency in basic skills is a necessary prerequisite to higher-level functioning in both reading and math. They suggest that children often do poorly in these subjects because they may have failed to master the subcomponent processes required to understand text and to solve math problems. A common example can be taken from reading.

Consider a child who cannot recognize words by sight. As he reads he must devote excessive attention to the task of word recognition by sounding out each word phoneme by phoneme. This cumbersome process leaves little room for attention to higher-level processes such as thinking about the meaning of words or sentences. When this occurs, comprehension is poor, and the child learns little from the reading material.

Problems similar to those encountered in reading also occur in math. If a student constantly has to compute the answers to basic facts, less of that student's thinking capacity can be devoted to higher-level concepts than a student who can effortlessly recall the answers to basic facts. For example, a child who is performing multiple-digit division must monitor constantly where he is in that procedure. If the child must use primitive counting strategies to subtract or multiply during the division process, the attention and memory resources devoted to these procedures reduce the student's ability to monitor and attend to the larger division problem. The result is that the student often fails to grasp the concepts involved in multiple-digit division.

Recent research in cognitive science, using functional magnetic resonance imaging (fMRI), has revealed the actual shift in brain activation patterns as untrained math facts are learned (Delazer et al., 2003). As predicted by Dehaene (1997, 1999, 2003), instruction and practice cause math fact processing to move from a quantitative area of the brain to one related to automatic retrieval. Delazer and her colleagues suggest that this shift aids the solving of complex computations that require "the selection of an appropriate resolution algorithm, retrieval of intermediate results, storage and updating in working memory" by substituting some of the intermediate steps with automatic retrieval (Delazer et al., 2004). The research cited above highlights the importance of math fact fluency; however, the computation capabilities of American students appear to be falling. Tom Loveless of the Brookings Institute has

reviewed responses to select items on the National Assessment of Educational Progress (NAEP) and concluded that performance on basic arithmetic facts declined in the 1990s (Loveless, 2003). Clearly, students need help to develop rapid, effortless, and errorless recall of basic math facts.

Mathematical Knowledge

Mathematical knowledge of basic facts can be classified into two categories. The first category, called *declarative knowledge*, can be conceptualized as an interrelated network of relationships containing basic problems and their answers, such as 4+7=11 or 11-4=7. The facts stored in this network have different "strengths" that determine how long it takes to retrieve an answer. The stronger the relationship, the more rapid and effortless is the retrieval process. For example, if the fact 2+3=5 has greater associative strength than the fact 7+5=12, it will take less time to retrieve the answer 5 to the first of these two problems (Pellegrino & Goldman, 1987).

Ideally, all the facts stored in this network are retrieved from memory quickly, effortlessly, and without error. However, this is often not the case with many students, particularly those with learning problems. These students, for a variety of reasons, have not established a declarative knowledge network; and instead of retrieving facts from memory, they rely on a second category of mathematics knowledge, called *procedural knowledge*.

Procedural knowledge refers to methods that can be used to derive answers for problems lacking pre-stored answers. For example, in the problem 6+8, a student might use a common "counting on" strategy in which the larger of the two addends (8) is stated and the student increments the smaller addend on his or her fingers while saying 9, 10, 11, 12, 13, 14. Although correct answers can be obtained using procedural knowledge, these procedures are effortful, slow, error-prone, and they appear to interfere with learning and understanding higher-order concepts.

Underlying both declarative and procedural knowledge in mathematics is a type of understanding typically called *number sense*. While several definitions of number sense can be found (see, for instance, NCTM Standards 2004 or Case 1998), academics generally agree that it involves an awareness of number names, values, and relationships. Children with number sense recognize the relative differences in number quantity and how those differences can be represented. Number sense gives meaning both to an automatic math fact and to a computational procedure. Gersten and Chard roughly compare the importance of number sense in computation to the need for phonemic awareness in reading (Gersten & Chard, 1999).

Both are critical building blocks. Garnett describes a typical hierarchy of procedures, or strategies, that rests upon number sense and leads eventually to automatic recall (Garnett, 1992). All elements—number sense, procedural knowledge, and declarative knowledge—must be developed together to achieve full math fact fluency.

Normal Development of Math Fact Fluency

Given the importance of the fluent recall of basic facts, the main concern is how this ability develops. For many children, at any point in time from preschool through at least the fourth grade, they will have some facts that can be retrieved from memory with little effort and some that need to be calculated using some counting strategy. From the fourth grade through adulthood, answers to basic math facts are recalled from memory with a continued strengthening of relationships between problems and answers that results in further increases in fluency (Ashcraft, 1985).

The acquisition of math facts generally progresses from a deliberate, procedural, and error-prone calculation to one that is fast, efficient, and accurate (Ashcraft, 1992; Fuson, 1982, 1988; Siegler, 1988). In a typical developmental path in addition, for instance, students begin adding using a strategy called "counting all" that gives way to a "counting on" strategy, which in turn gives way to linking new facts to known facts (Garnett, 1992). In multiplication, a student might employ repeated addition or skip counting as initial procedures for calculating the facts (Siegler, 1988). With repeated exposures, most normally developing students establish a memory relationship with each fact. Instead of calculating it, they recall it automatically.

Strategy	Representative use to solve 2+4
Counting all	"1, 2…1, 2, 3, 4…1, 2, 3, 4, 5, 6"
Shortcut sum	"1, 2, 3, 4, 5, 6"
Finger display	"Displays 2 fingers, then 4 fingers; says 6"
Counting on from the first addend	"2…3, 4, 5, 6" or "3, 4, 5, 6"
Counting on from the larger addend	"4…5, 6" or "5, 6"
Linking	"2 + 2 = 4, + 2 more = 6"
Retrieval	"6"

In contrast, most math-delayed children, along with those who have never received systematic math fact instruction, show a serious problem with respect to the retrieval of elementary number facts. Fleischner, Garnett, and Shepard (1982), as well as Hasselbring, Goin, and Bransford (1988), have found that

learning-disabled children are substantially less proficient than their non-disabled peers in retrieving the answers to basic math facts in addition and subtraction. Although information is still emerging about the particular difficulties experienced by these children in the retrieval of this information, the evidence that does exist suggests that these children do not suffer from a conceptual deficit (Russell & Ginsburg, 1984), but rather from some sort of disruption to normal development of their network of relationships between facts and answers. That is, these students often have well-developed number sense and procedural knowledge—they can figure out the answer to any fact given enough time. But because they have poorly developed declarative knowledge, they have minimal ability to recall anything but the most basic facts from memory. What this suggests is that there are huge differences in the amount of instruction individual children need to become fluent at retrieving answers to basic math facts.

As shown in Figure 1, by age seven, non-math-delayed students can recall more facts from memory than their math-delayed peers. Further, this discrepancy increases as age increases. As math-delayed students get older, they fall further and further behind their non-math-delayed peers in the ability to recall basic math facts from memory (Hasselbring et al., 1988). In addition, this lack of fluency interferes with the development of higher-order mathematical thinking and problem solving.
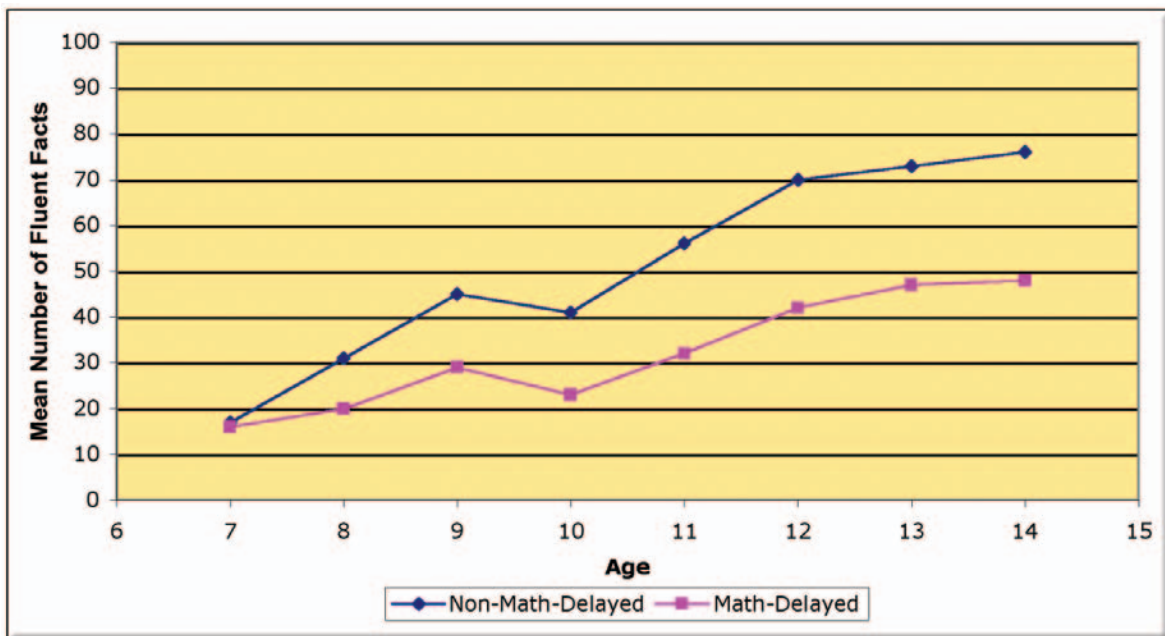

Figure 1. A comparison of the mean number of fluent addition facts by age for Non-Math-Delayed and Math-Delayed students

Developing Fluency in Math-Delayed Children

To counteract the problem described above, the FASTT approach has been used successfully to develop mathematical fluency. It appears that the key to making the retrieval of basic math facts fluent is to first establish a mental link between the facts and their answers. *FASTT Math* embodies several unique design features to help develop these relationships. These features include:

- Identification of fluent and non-fluent facts
- Restricted presentation of nonü-fluent information
- Student generation of problem/answer pairs
- Use of controlled response times
- Spaced presentation of non-fluent information
- Appropriate use of drill and practice

Each of the features listed above (and described in more detail below) adds to the effectiveness of the program.

IDENTIFICATION OF FLUENT AND NON-FLUENT FACTS

At any given point in time, most students recall some facts automatically; they answer others using counting or other non-automatic strategies. Drill and practice programs have demonstrated a positive effect on improving the retrieval speed for facts already being recalled from memory. However, drill and practice had no effect on developing automaticity for non-recalled facts (Hasselbring, Goin, & Sherwood, 1986). Consequently, to facilitate the automatic recall of all facts, instruction must be focused on non-automatized facts while practice and review are given on facts that are already being recalled from memory. Thus identifying and separating fluent from non-fluent facts is important.

FASTT Math begins with a computer-based assessment that presents all the basic facts in an operation and records the amount of time that the child takes to answer each fact correctly. By measuring the latencies of student responses, the program can accurately determine the facts that are being recalled from memory and those that are solved using a counting strategy. (Note: Response latency is determined by measuring the time difference between simply typing the number 21 and typing the answer when presented with the multiplication fact 7x3.)

Following this initial placement quiz, *FASTT Math* constructs a fact grid, as shown below. The grid allows the student (and teacher) to visually see the fluent ("Fast") facts and those that the student answered slowly or incorrectly ("Study" facts). The grid shown here indicates a common pattern in many math-delayed students. This student has automatized most of the facts that include 0 and 1 as the minimum addend (e.g., 0+0 to 0+9, and 1+1 to 1+8, and the reversals). Also, he has automatized a few facts with 2 as a minimum addend, and some of the doubles (e.g., 3+3, 4+4, and 5+5).
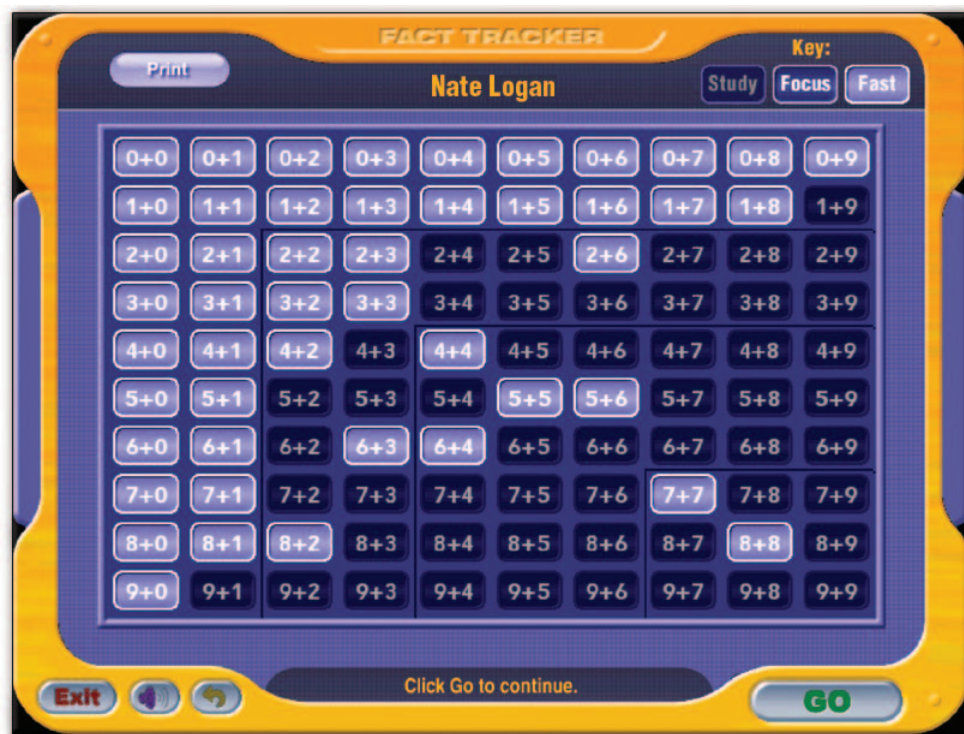


Fact Grid screen for a typical math-delayed student

RESTRICTED PRESENTATION OF NON-FLUENT INFORMATION

The program expands the student's declarative knowledge network by building on existing knowledge. Consider the student whose fact grid is shown in the image above. The *FASTT Math* software would begin instruction on the facts 1+9 and 9+1, because the student already has automatized all other facts with a minimum addend of 1. As a general rule, the program selects facts to be automatized based upon the size of the minimum addend. For example, once all facts with a minimum addend of 1 have been automatized, *FASTT Math* begins to select facts with a minimum addend of 2, and so on, until all the "2s" have been automatized.
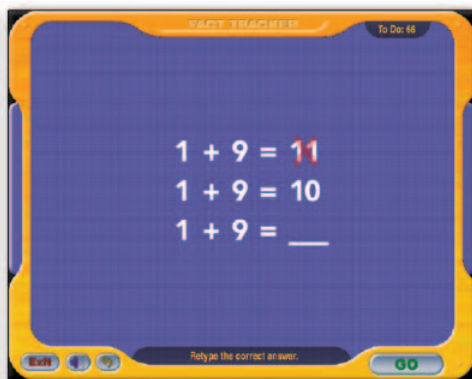
The research (Hasselbring et al., 1988) suggests that it is best to work on developing this declarative knowledge by focusing on a very small set of new target facts at any one time—no more than two facts and their reversals. Instruction on this target set continues until the student can retrieve the answers to the two new facts consistently and without using counting strategies.

STUDENT GENERATION OF PROBLEM/ANSWER PAIRS

Recent fMRI studies of math fact recall (Dehaene et al., 1999; Delazer et al., 2004) suggest that automatically retrieved facts are stored in the same region that houses word associations, suggesting a potential linguistic relationship between the calculation (eight times seven) and its answer (fifty-six). In order to construct a memory relationship of this type, *FASTT Math* explicitly requires students to type each newly introduced fact. By generating the problem/answer pair, students connect the two elements together. And when students falter in holding that connection in memory, the program demands that they retype the fact to reestablish the relationship.
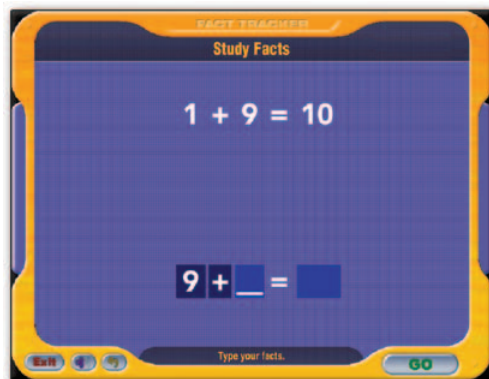
*Typing a newly introduced fact*   *Retyping a fact to reestablish the relationship*

USE OF CONTROLLED RESPONSE TIMES

Once a problem/answer relationship is established, *FASTT Math* uses controlled response times to reinforce the memory connection and inhibit the use of counting or other non-automatic strategies. A controlled response time is the amount of time allotted to retrieve and provide the answer to the fact. *FASTT Math* begins with a controlled response time of 1.25 seconds, forcing students to abandon inefficient strategies and to retrieve answers rapidly from the declarative knowledge network.

If the controlled response time lapses before the child can respond, or if the student answers incorrectly, the program provides corrective feedback by presenting the problem/answer relationship again. This continues until the child gives the correct answer within the controlled response time.

SPACED PRESENTATION OF NON-FLUENT INFORMATION

FASTT Math develops a declarative knowledge network by interspersing the two new "target" facts with other already automatized facts in a pre-specified, expanding order. Each time the target fact is presented, another automatized fact is added as a "spacer" so that the amount of time between presentations of the target fact is expanded. This "expanding recall" model requires the student to retrieve the correct answers to the target facts over longer and longer periods.
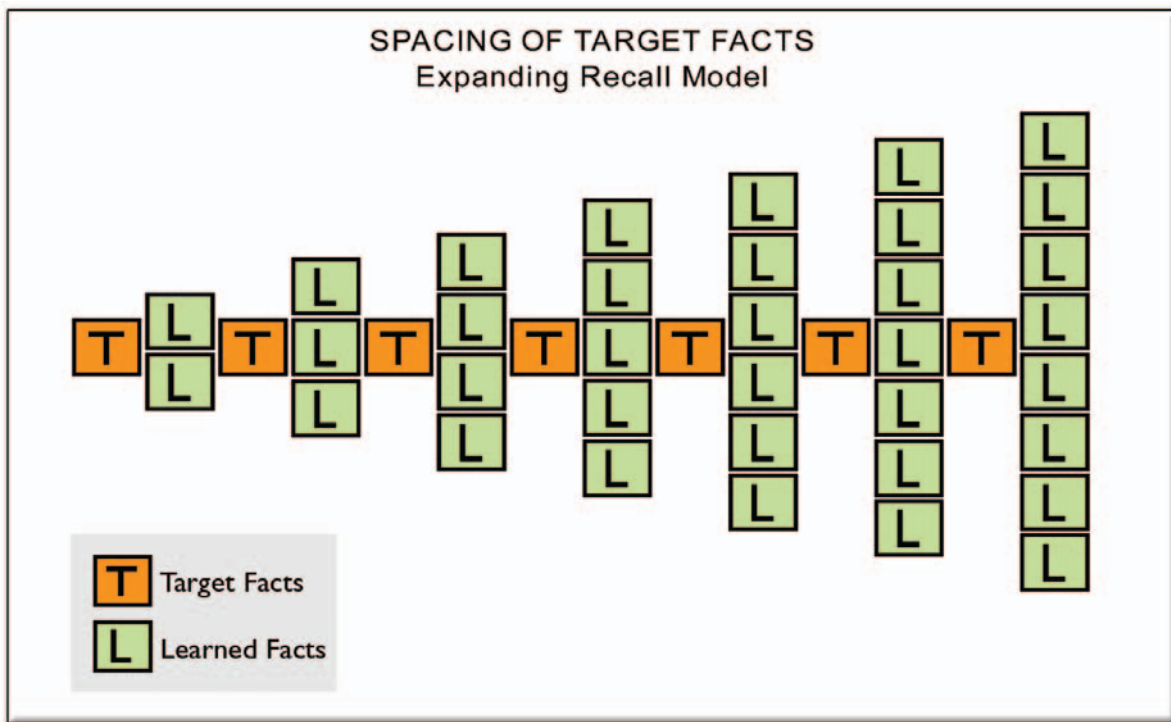
Presenting non-fluent facts interspersed with previously mastered facts

Only after a student is consistently able to retrieve the answer to a target fact within the controlled response time is that fact added to the child's set of drill and practice facts. Drill and practice has been shown to be effective only with facts that are already being retrieved from memory. *FASTT Math* systematically builds a memory relationship before it reinforces speed of recall with appropriate drill and practice activities.

Effectiveness of the FASTT Math Approach

The principles embodied in *FASTT Math* have been validated over several years of research with more than 400 students. This research with math-delayed children has shown that the *FASTT Math* approach can be extremely powerful for developing fluency with basic math facts. Generally, the findings show that when used daily, for about ten minutes, most math-delayed children can develop fluency with all basic facts in a single operation after approximately 100 sessions. The key to success appears to lie in the consistent use of the program. As expected, students who use the program regularly do much better than students who are only occasional users.

As shown in Figure 2, the effects of using the *FASTT Math* approach can be quite striking. In a study conducted by Hasselbring and Goin (1988), three groups of students were matched for age, sex, and race. Two of the groups consisted of math-delayed students and the remaining group consisted of non-math-delayed students. In the experiment, one of the math-delayed groups (Math-Delayed Experimental) received an average of 54 ten-minute sessions on the software program for addition; the other two groups (Non-Math-Delayed and Math-Delayed Contrast) received only traditional fluency instruction delivered by their classroom teachers. As the data shows, the math-delayed students receiving instruction with the *FASTT Math* approach gained, on the average, 19 new fluent facts while their math-delayed peers receiving traditional instruction gained no new facts and their non-math-delayed peers gained only 7 new facts. Perhaps more impressive are the maintenance data. When the experimental students were tested four months after the post-test following summer vacation, the students regressed by only 6 facts, indicating that once facts become fluent through this method, they are retained at a high level.
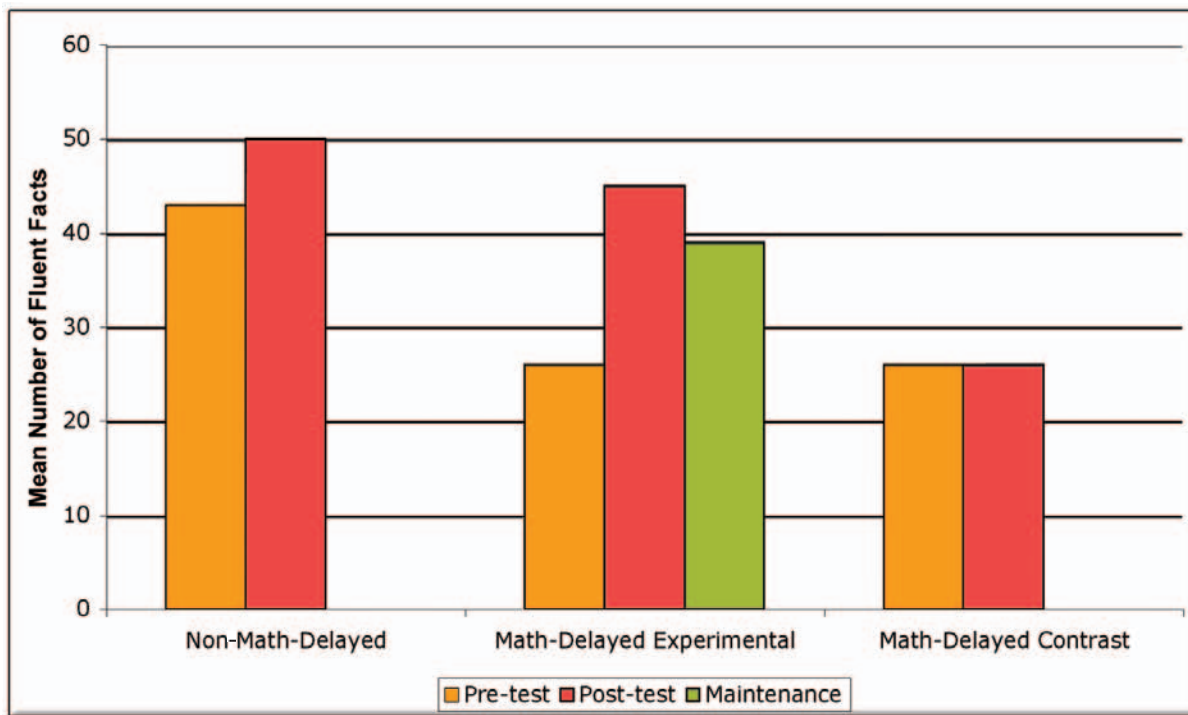
Figure 2. A comparison of the mean number of fluent addition facts for Non-Math-Delayed and Math-Delayed students

The results of this experiment have been replicated several times across all four operations. In all cases, when used consistently, the *FASTT Math* approach has a positive effect on developing mathematical fluency in both math-delayed and non-math-delayed students. Although *FASTT Math* is effective for all students needing assistance with developing fact fluency, it appears to be especially effective for students labeled as at-risk and learning disabled.

REFERENCES

Ashcraft, M.H. (1985). *Children's knowledge of simple arithmetic: A developmental model and simulation.* Unpublished manuscript, Cleveland State University.

Ashcraft, M.H. Cognitive arithmetic: A review of data and theory. *Cognition* 44 (1992), 75–106.

Case, Robbie. (April 1998). *A Psychological Model of Number Sense and Its Development.* Stanford University and the University of Toronto.

Dehaene, S., Piazza, M., Pinel, P., Cohen, L. (2003) Three parietal circuits for number processing. *Cognitive Neuro ychology.* Vol. 20, nos. 3–6, pp. 487–506.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., Tsivkin, S. Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science* 284 (May 7, 1999) 284, 970–974.

Dehaene, S. *The Number Sense: How the Mind Creates Mathematics.* New York: Oxford University Press, 1997.

Delazer, M., Domahs, F., Bartha, L., Brenneis, C., Locky, A., Trieb, T. (2004). The acquisition of arithmetic knowledge—an fMRI study. *Cortex* 40 (2004), 166–167.

Delazer, M., Domahs, F., Bartha, L., Brenneis, C., Locky, A., Trieb, T. Benke, T., (2003). Learning complex arithmetic—an fMRI study. *Cognitive Brain Research* 18 (2003), 76–88.

Fleischner, J.E., Garnett, K., & Shepard, M.J. (1982). Proficiency in arithmetic basic facts computation of learning disabled children. *Focus on Learning Problems in Mathematics* 4, 47–56.

Fuson, K.C. (1982). "An analysis of the counting-on procedure in addition." In T.H. Carpenter, J.M. Moser, T.H. Romberg, (Eds.), *Addition and Subtraction: A Cognitive Perspective.* Hillsdale, NJ: Lawrence Erlbaum, pp. 67–78.

Fuson, K.C. *Children's Counting and Concepts of Number.* New York: Springer, 1988.

Garnett, K. Math learning disabilities. *Division for Learning Disabilities Journal of CEC.* November, 1998.

Garnett, K. Developing fluency with basic number facts: Intervention for students with learning disabilities. *Learning Disabilities Research & Practice* (1992), 7:210–216.

Geary, D.C., Hoard, M.K. Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia. *Aphasiology* (2001), 15 (7), 635–647.

Gersten, R. & Chard, D. Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education* (1999), 3, 18–29.

REFERENCES - *continued*

Hasselbring, T.S., Goin, L., & Bransford, J.D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children* 20, 1–7.

Hasselbring, T.S., Goin, L., & Sherwood, R.D. (1986). "The effects of computer-based drill and practice on automaticity." Technical report. Nashville, TN: Vanderbilt University, Learning Technology Center.

LaBerge, D. & Samuels, S. (1974). Toward a theory of automatic information processing in reading. *Cognitive Psychology* 6, 293–323.

Lesgold, A.M. (1983). "A rationale for computer based reading instruction." In A. Wilkinson, (Ed.). *Classroom computers and cognitive science*. New York: Academic Press.

Loveless, T. "Trends in Math Achievement: The Importance of Basic Skills." Presentation at the Mathematics Summit, Washington, DC: February 6, 2003.

Pellegrino, J.W. & Goldman, S.R. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities* 20, 23–32, 57.

Resnick, L.B. (1983). "A development theory of number understanding." In Herbert P. Ginsburg (Ed.), *The Development of Mathematical Thinking.* New York: Academic Press, pp. 109–151.

Royer, J.M., Tronsky, L.N., Chan, Y., Jackson, S.J., & Marchant, H. (1999). Math fact retrieval as the cognitive mechanism underlying gender differences in math test performance. *Contemporary Educational Psychology* 24, 181–266.

Russell, R.L. & Ginsburg, H.P. (1984). Cognitive analysis of children's mathematics difficulties. *Cognition & Instruction* 1(2), 217–244.

Siegler, R.S. Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology*, Gen. 117 (1988), 258–275.

Torgesen, J.K. (1984). Instructional Use of Microcomputers with Elementary Aged Mildly Handicapped Children. *Microcomputers and Exceptional Children*. 1, 37-48.

Whitehurst, G. IES Director's presentation at the Mathematics Summit, Washington, DC: February 6, 2003.

Woodward, J. & Baxter, J. (1997). The effects of an innovative approach to mathematics on academically low-achieving students in inclusive settings. *Focus on Exceptional Children* 63, 373–388.

ABOUT TOM SNYDER PRODUCTIONS & SCHOLASTIC

About Tom Snyder Productions

Tom Snyder Productions, Inc., a Scholastic company, is a leading developer and publisher of educational software for K-12 classrooms. The company was founded over 20 years ago by Tom Snyder, a former science and music teacher who pioneered the utilization of technology in the classroom to improve student understanding and performance. Today, Tom Snyder Productions has received over 150 prestigious industry awards and its products are used in over 400,000 classrooms. The company's software titles cover each curriculum area, and its professional development team has helped more than 175,000 teachers learn to integrate technology effectively into their curricula. (www.tomsnyder.com)

About Scholastic

Scholastic Corporation (NASDAQ: SCHL) is the world's largest publisher and distributor of children's books and a leader in educational technology. Scholastic creates quality educational and entertaining materials and products for use in school and at home, including children's books, magazines, technology-based products, teacher materials, television programming, film, videos, and toys. The Company distributes its products and services through a variety of channels, including proprietary school-based book clubs, school-based book fairs, and school-based and direct-to-home continuity programs; retail stores, schools, libraries and television networks; and the Company's Internet site, www.scholastic.com.

Research Foundation
Paper

Scholastic
Research &
Results

Tom Snyder Productions
www.tomsnyder.com
800-342-0236