Educational Leadership
November 2007

Nine Ways to Catch Kids Up

How do we help floundering students who lack basic math concepts?

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Paul, a 4th grader, was struggling to learn multiplication. Paul's teacher was concerned that he typically worked very slowly in math and "didn't get much done." I agreed to see whether I could figure out the nature of Paul's difficulty. Here's how our conversation began:

MARILYN: Can you tell me something you know about multiplication?

PAUL: [*Thinks, then responds*] 6 x 8 is 48.

MARILYN: Do you know how much 6 x 9 is?

PAUL: I don't know that one. I didn't learn it yet.

MARILYN: Can you figure it out some way?

PAUL: [*Sits silently for a moment and then shakes his head.*]

MARILYN: How did you learn 6 x 8?

PAUL: [*Brightens and grins*] It's easy—goin' fishing, got no bait, 6 x 8 is 48.

As I talked with Paul, I found out that multiplication was a mystery to him. Because of his weak foundation of understanding, he was falling behind his classmates, who were multiplying problems like 683 x 4. Before he could begin to tackle such problems, Paul needed to understand the concept of multiplication and how it connects to addition.

Paul wasn't the only student in this class who was floundering. Through talking with teachers and drawing on my own teaching experience, I've realized that in every class a handful of students are at serious risk of failure in mathematics and aren't being adequately served by the instruction offered. What should we do for such students?

Grappling with Interventions

My exchange with Paul reminded me of three issues that are essential to teaching mathematics:

- It's important to help students make connections among mathematical ideas so they do not see these ideas as disconnected facts. (Paul saw each multiplication fact as a separate piece of information to memorize.)
- It's important to build students' new understandings on the foundation of their prior learning. (Paul did not make use of what he knew about addition to figure products.)
- It's important to remember that students' correct answers, without accompanying explanations of how they reason, are not sufficient for judging mathematical understanding. (Paul's initial correct answer about the product of 6 x 8 masked his lack of deeper understanding.)

For many years, my professional focus has been on finding ways to more effectively teach arithmetic, the cornerstone of elementary mathematics. Along with teaching students basic numerical concepts and skills, instruction in number and operations prepares them for algebra. I've developed lessons that help students make sense of number and operations with attention to three important elements—computation, number sense, and problem solving. My intent has been to avoid the "yours is not to question why, just invert and multiply" approach and to create lessons that are accessible to all students and that teach skills in the context of deeper understanding. Of course, even well-planned lessons will require differentiated instruction, and much of the differentiation needed can happen within regular classroom instruction.

But students like Paul present a greater challenge. Many are already at least a year behind and lack the foundation of mathematical understanding on which to build new learning. They may have multiple misconceptions that hamper progress. They have experienced failure and lack confidence.

Such students not only demand more time and attention, but they also need supplemental instruction that differs from the regular program and is designed specifically for their success. I've recently shifted my professional focus to thinking about the kind of instruction we need to serve students like Paul. My colleagues and I have developed lessons that provide effective interventions for teaching number and operations to those far behind. We've grappled with how to provide instruction that is engaging, offers scaffolded instruction in bite-sized learning experiences, is paced for students' success, provides the practice students need to cement fragile understanding and skills, and bolsters students' mathematical foundations along with their confidence.

Extra help for struggling learners must be more than additional practice.

In developing intervention instruction, I have reaffirmed my longtime commitment to helping students learn facts and skills—the basics of arithmetic. But I've also reaffirmed that "the basics" of number and operations for all students, including those who struggle, must address all three aspects of numerical proficiency—computation, number sense, and problem solving. Only when the basics include understanding as well as skill proficiency will all students learn what they need for their continued success.

Essential Strategies

I have found the following nine strategies to be essential to successful intervention instruction for struggling math learners. Most of these strategies will need to be applied in a supplementary setting, but teachers can use some of them in large-group instruction.

1. Determine and Scaffold the Essential Mathematics Content

Determining the essential mathematics content is like peeling an onion—we must identify those concepts and skills we want students to learn and discard what is extraneous. Only then can teachers scaffold this content, organizing it into manageable chunks and sequencing these chunks for learning.

For Paul to multiply 683 x 4, for example, he needs a collection of certain skills. He must know the basic multiplication facts. He needs an understanding of place value that allows him to think about 683 as 600 + 80 + 3. He needs to be able to apply the distributive property to figure and then combine partial products. For this particular problem, he needs to be able to multiply 4 by 3 (one of the basic facts); 4 by 80 (or 8 x 10, a multiple of 10); 4 and by 600 (or 6 x 100, a multiple of a power of 10). To master multidigit multiplication, Paul must be able to combine these skills with ease. Thus, lesson planning must ensure that each skill is explicitly taught and practiced.

2. Pace Lessons Carefully

We've all seen the look in students' eyes when they get lost in math class. When it appears, ideally teachers should stop, deal with the confusion, and move on only when all students are ready. Yet curriculum demands keep teachers pressing forward, even when some students lag behind. Students who struggle typically need more time to grapple with new ideas and practice new skills in order to internalize them. Many of these students need to unlearn before they relearn.

3. Build in a Routine of Support

Students are quick to reveal when a lesson hasn't been scaffolded sufficiently or paced slowly enough: As soon as you give an assignment, hands shoot up for help. Avoid this scenario by building in a routine of support to reinforce concepts and skills before students are expected to complete independent work. I have found a four-stage process helpful for supporting students.

In the first stage, the teacher models what students are expected to learn and records the appropriate mathematical representation on the board. For example, to simultaneously give students practice multiplying and experience applying the associative and commutative properties, we present

My "Aha!" Moment

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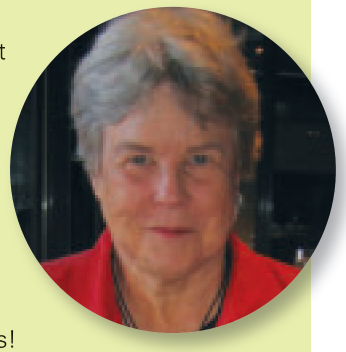
My "aha" moment came long after I had finished a masters in mathematics, taught mathematics in secondary school and college, and completed a doctorate in mathematics education. Although I enjoyed the rigor of learning and applying rules, mathematics was more like a puzzle than an elegant body of knowledge.



Many years of work on a mathematics program for elementary schools led to that moment. I realized that mathematics was more than rules—even the beginnings of mathematics were interesting. Working with elementary students and teachers, I saw that students could make sense of basic mathematical concepts and procedures, and teachers could help them do so. The teachers also posed problems to move students forward, gently let them struggle, and valued their approaches. What a contrast to how I had taught and learned mathematics!

With vivid memories of a number-theory course in which I memorized the proofs to 40 theorems for the final exam, I cautiously began teaching a number-theory course for prospective middle school teachers. My aha moment with these students was a semester long. We investigated number-theory ideas, I made sense of what I had memorized, and my students learned along with me. My teaching was changed forever.

them with problems that involve multiplying three one-digit factors. An appropriate first problem is 2 x 3 x 4. The teacher thinks aloud to demonstrate three ways of working this problem. He or she might say,

> I could start by multiplying 2 x 3 to get 6, and then multiply 6 x 4 to get 24. Or I could first multiply 2 x 4, and then multiply 8 x 3, which gives 24 again. Or I could do 3 x 4, and then 12 x 2. All three ways produce the same product of 24.

As the teacher describes these operations, he or she could write on the board:

2 x 3 x 4	2 x 3 x 4	2 x 3 x 4
6 x 4 = 24	8 x 3 = 24	2 x 12 = 24

It's important to point out that solving a problem in more than one way is a good strategy for checking your answer.

In the second stage, the teacher models again with a similar problem—such as 2 x 4 x 5—but this time elicits responses from students. For example, the teacher might ask, "Which two factors might you multiply first? What is the product of those two factors? What should we multiply next? What is another way to start?" Asking such questions allows the teacher to reinforce correct mathematical vocabulary. As students respond, the teacher again records different ways to solve the problem on the board.

During the third stage, the teacher presents a similar problem—for example, 2 x 3 x 5. After taking a moment to think on their own, students work in pairs to solve the problem in three different ways, recording their work. As students report back to the class, the teacher writes on the board and discusses their problem-solving choices with the group.

In the fourth stage, students work independently, referring to the work recorded on the board if needed. This routine both sets an expectation for student involvement and gives learners the direction and support they need to be successful.

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4. Foster Student Interaction

We know something best once we've taught it. Teaching entails communicating ideas coherently, which requires the one teaching to formulate, reflect on, and clarify those ideas—all processes that support learning. Giving students opportunities to voice their ideas and explain them to others helps extend and cement their learning.

Thus, to strengthen the math understandings of students who lag behind, make student interaction an integral part of instruction. You might implement the *think-pair-share* strategy, also called *turn and talk*. Students are first asked to collect their thoughts on their own, and then talk with a partner; finally, students share their ideas with the whole group. Maximizing students' opportunities to express their math knowledge verbally is particularly valuable for students who are developing English language skills.

5. Make Connections Explicit

Students who need intervention instruction typically fail to look for relationships or make connections among mathematical ideas on their own. They need help building new learning on what they already know. For example, Paul needed explicit instruction to understand how thinking about 6 x 8 could give him access to the solution for 6 x 9. He needed to connect the meaning of multiplication to what he already knew about addition (that 6 x 8 can be thought of as combining 6 groups of 8). He needed time and practice to cement this understanding for all multiplication problems. He would benefit from investigating six groups of other numbers—6 x 2, 6 x 3, and so on—and looking at the numerical pattern of these products. Teachers need to provide many experiences like these, carefully sequenced and paced, to prepare students like Paul to grasp ideas like how 6 x 9 connects to 6 x 8.

6. Encourage Mental Calculations

Calculating mentally builds students' ability to reason and fosters their number sense. Once students have a foundational understanding of multiplication, it's key for them to learn the basic multiplication facts—but their experience with multiplying mentally should expand beyond these basics. For example, students should investigate patterns that help them mentally multiply any number by a power of 10. I am concerned when I see a student multiply 18 x 10, for example, by reaching for a pencil and writing:

$$\begin{array}{r} 18 \\ \times\,10 \\ \hline 00 \\ 18 \\ \hline 180 \end{array}$$

Revisiting students' prior work with multiplying three factors can help

develop their skills with multiplying mentally. Helping students judge which way is most efficient to multiply three factors, depending on the numbers at hand, deepens their understanding. For example, to multiply 2 x 9 x 5, students have the following options:

2 x 9 x 5	2 x 9 x 5	2 x 9 x 5
18 x 5 = 90	10 x 9 = 90	2 x 45 = 90

Guiding students to check for factors that produce a product of 10 helps build the tools they need to reason mathematically.

When students calculate mentally, they can estimate before they solve problems so that they can judge whether the answer they arrive at makes sense. For example, to estimate the product of 683 x 4, students could figure out the answer to 700 x 4. You can help students multiply 700 x 4 mentally by building on their prior experience changing three-factor problems to two-factor problems: Now they can change a two-factor problem—700 x 4—into a three-factor problem that includes a power of 10—7 x 100 x 4. Encourage students to multiply by the power of 10 last for easiest computing.

7. Help Students Use Written Calculations to Track Thinking

Students should be able to multiply 700 x 4 in their heads, but they'll need pencil and paper to multiply 683 x 4. As students learn and practice procedures for calculating, their calculating with paper and pencil should be clearly rooted in an understanding of math concepts. Help students see paper and pencil as a tool for keeping track of how they think. For example, to multiply 14 x 6 in their heads, students can first multiply 10 x 6 to get 60, then 4 x 6 to get 24, and then combine the two partial products, 60 and 24. To keep track of the partial products, they might write:

14 x 6
10 x 6 = 60
4 x 6 = 24
60 + 24 = 84

They can also reason and calculate this way for problems that involve multiplying by three-digit numbers, like 683 x 4.

8. Provide Practice

Struggling math students typically need a great deal of practice. It's essential that practice be directly connected to students' immediate learning experiences. Choose practice problems that support the elements of your scaffolded instruction, always promoting understanding as well as skills. I recommend giving assignments through the four-stage support routine, allowing for a gradual release to independent work.

Games can be another effective way to stimulate student practice. For example, a game like *Pathways* (see Figure 1 for a sample game board and instructions) gives students practice with multiplication. Students hone multiplication skills by marking boxes on the board that share a common side and that each contain a product of two designated factors.

9. Build In Vocabulary Instruction

The meanings of words in math—for example, *even*, *odd*, *product*, and *factor*—often differ from their use in common language. Many students needing math intervention have weak mathematical vocabularies. It's key that students develop a firm understanding of mathematical concepts before learning new vocabulary, so that they can anchor terminology in their understanding. We should explicitly teach vocabulary in the context of a learning activity and then use it consistently. A math vocabulary chart can help keep both teacher and students focused on the importance of accurately using math terms.

When Should We Offer Intervention?

There is no one answer to when teachers should provide intervention instruction on a topic a particular student is struggling with. Three different timing scenarios suggest themselves, each with pluses and caveats.

While the Class Is Studying the Topic
Extra help for struggling learners must be more than additional practice on the topic the class is working on. We must also provide comprehensive instruction geared to repairing the student's shaky foundation of understanding.

■ *The plus*: Intervening at this time may give students the support they need to keep up with the class.

■ *The caveat*: Students may have a serious lack of background that requires reaching back to mathematical concepts taught in previous years. The focus should be on the underlying math, not on class assignments. For example, while others are learning multidigit multiplication, floundering students may need experiences to help them learn basic underlying concepts, such as that 5 x 9 can be interpreted as five groups of nine.

Before the Class Studies the Topic
Suppose the class is studying multiplication but will begin a unit on fractions within a month, first by cutting out individual fraction kits. It would be extremely effective for at-risk students to have the fraction kit experience before the others, and then to experience it again with the class.

■ *The plus*: We prepare students so they can learn *with* their classmates.

■ *The caveat*: With this approach, struggling students are studying two different and unrelated mathematics topics at the same time.

After the Class Has Studied the Topic.
This approach offers learners a repeat experience, such as during summer school, with a math area that initially challenged them.

■ *The plus*: Students get a fresh start in a new situation.

■ *The caveat*: Waiting until after the rest of the class has studied a topic to intervene can compound a student's confusion and failure during regular class instruction.

Many students needing math intervention have weak mathematical vocabularies.

FIGURE 1. Pathways Multiplication Game

72	36	49	88	54
84	77	96	132	56
63	81	48	108	121
66	99	144	64	42

6 7 8 9 **11** 12

Player 1 chooses two numbers from those listed (in the game shown here, 6 and 11) and circles the product of those two numbers on the board with his or her color of marker.

Player 2 changes just one of the numbers to another from the list (for example, changing 6 to 9, so the factors are now 9 and 11) and circles the product with a second color.

Player 1 might now change the 11 to another 9 and circle 81 on the board. Play continues until one player has completed a continuous pathway from one side to the other by circling boxes that share a common side or corner. To support intervention students, have pairs play against pairs.

How My Teaching Has Changed

Developing intervention lessons for at-risk students has not only been an all-consuming professional focus for me in recent years, but has also reinforced my belief that instruction—for all students and especially for at-risk students—must emphasize understanding, sense making, and skills.

Thinking about how to serve students like Paul has contributed to changing my instructional practice. I am now much more intentional about creating and teaching lessons that help intervention students catch up and keep up, particularly scaffolding the mathematical content to introduce concepts and skills through a routine of support. Such careful scaffolding may not be necessary for students who learn mathematics easily, who know to look for connections, and who have mathematical intuition. But it is crucial for students at risk of failure who can't repair their math foundations on their own. EL

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Item 152684
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